

Image Reconstruction for Tomography Using Wavelet Transform

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Abstract— The reconstruction of tomography imaging is often corrupted by means of a number of measurements, projection data, noise in measurement, computation time, resolution and prior knowledge. This paper presents a new approach called wavelet based image reconstruction for tomography by using different thresholding methods are investigated in the presence of choosing different wavelet filters. The quality of the reconstructed image is expressed in terms of mean square error (MSE) as compared to the original image and found as improved compared to other techniques.

Index Terms—Computer Tomography, Decomposition, Image Reconstruction, Hard Threshold, MSE, Soft Thresholding, Wavelet Transform

1. INTRODUCTION

The reconstruction of images may expressed as the general problem of estimating a two dimensional object from a degraded version of this object. In Image reconstruction, observations result from the interaction between the unknown object and some scattering wave. In fields of tomography, lots of problems are faced in recovering original images from incomplete, indirect and noisy images. Images get corrupted during acquisition by camera sensors, receivers, environmental conditions, improper lightning, undesirable view angle etc., Also the problem of recovering an original image from noisy image has received an ever increasing attention in recent years.

In Computer Tomography, the reconstruction of image by Direct Fourier techniques is based on projection theory [1]. It has been shown that the class of linear edge detection operators used on images can be used for detection of edges directly from projection data [2]. This will help to reduce the burden of computation in recovering high resolution image. In edge preserving image reconstruction, the image is modeled as space variant prior distribution. An anisotropic diffusion works well in extracting boundaries for images which comprise smooth regions separated by discontinuities.

This boundary information is incorporated into the reconstruction algorithm [3]. In limited angle tomography, it requires given some approximate a priori knowledge. This will help to detect the knowledge of the locations and approximate magnitudes of some of the edges that lie parallel to the missing view angles [4].

The tomography reconstruction is ill posed problem where the projection data are noisy and incomplete data [5]. The performance of two types of analytical algorithms were discussed to reconstruct the tomography images, that is, the filtered backprojection algorithm and the backprojection filtering algorithm [6]. The multiresolution analysis is used for the image reconstruction in severely incomplete projection data [7].

The Natural Pixel representation is a matrix based reconstruction techniques for an incomplete data [8]. They incorporate the wavelet transform and total variation based penalties in to the maximum likelihood estimation and compare the results with EM algorithm and conjugate barrier algorithm [9]. The projection image is available from a limited angle of view discussed [10]. The Sinusoidal Hough Transform applied to tomography sensors which provide adequate quality of imaging from limited number of measurement [11]. An optimum wavelet based smoothing function incorporate in reconstruction process [12].

The quality of the generated tomograms could be improved by including a thresholding procedure in the reconstruction process for the purpose of minimizing distortions [13]. Based on threshold, the wavelet and wavelet packet decomposition involved in reconstruction [14]. The denoising method is used for removal of poisson noise in the projection data [15]. The Wavelet based approach used in the reconstruction of diffraction tomography [16].

The properties of wavelets to localize the Radon transform and can be used to reconstruct a local region of the cross section of object, using almost completely local data that significantly reduces the amount of exposure and computations in X-ray tomography [17]. The near radial quincunx multi resolution scheme is proposed [18].

The wavelet based algorithm into filtered back projection is used for image reconstruction [19]. The model based tomography reconstruction, which is based on wavelet packet representation of image and acquired projection data is presented [20].

2. DISCRETE WAVELET TRANSFORM

Assume the observed signal

$$y(t) = s(t) + n(t) \quad (1)$$

where $y(t)$, $s(t)$ and $n(t)$ are called observed signal, the original signal $s(t)$ with additive noise $n(t)$ as functions of time t

to be sampled. Let $W(\cdot)$ and $W^{-1}(\cdot)$ denote the forward and inverse wavelet transform. Let $D(\cdot, \lambda)$ denote the denoising operator with threshold λ . They intend to wavelet de noise $y(t)$ in order to recover $s^{\wedge}(t)$ as an estimate of $S(t)$.

$$S = w(y) \tag{1a}$$

$$Z = D(y, \lambda) \tag{1b}$$

$$S = w^{-1}(Z) \tag{1c}$$

Similar to the Fourier series expansion, the DWT maps a continuous variable $\Psi(t)$ into a sequence of coefficients, the resultant coefficients are called discrete wavelet transform of $\Psi(t)$. Its representation involves the decomposition of the signals in wavelet basis function $\Psi(t)$ given by

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} [(t-b)/a] \tag{2}$$

where a, b are called scale and position parameters as respectively.

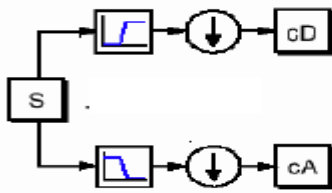


Fig. 1: Single Level Wavelet Decomposition (Analysis)

The multi resolution analysis is given by S. Mallat and Mayer proves that any conjugate mirror filter characterizes a wavelet Ψ .

The wavelet decomposition of a signal $x(t)$ based a multi resolution theory can be obtained using filter. The filter based wavelet decomposition is shown in Fig. 1. The above arrangement has used two wavelet decomposition filters which are high pass and low pass respectively followed by down sampling by 2 producing half input data point of high and low frequency. The high frequency coefficients (CD) and low frequency coefficients are called approximate coefficients (CA). The signal can be reconstructed back by inverse wavelet transform. The corresponding filter bank structure for reconstruction is shown in Fig 2.

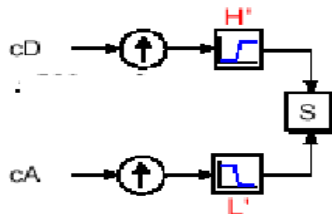


Fig. 2: Single Level Wavelet Reconstruction (Synthesis)

The above arrangement has used two wavelet reconstruction filters which are up sampling by 2 followed by high pass and low pass filters producing the original signal S. The reconstruction process called inverse wavelet trans-

form.

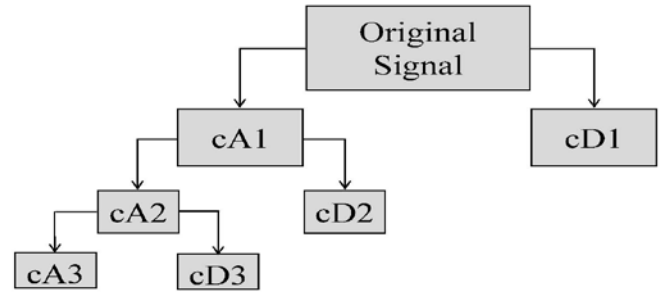


Fig. 3: Three Level Wavelet Decomposition Tree

The signal $S(t)$ can be de composed into several levels. A three level wavelet decomposition tree is shown in Fig 3 Wavelet reconstruction scheme: let us assume signal $S(t)$ is corrupted by noise $n(t)$ as shown in Eq. 1 where $n(t)$ is white noise. The wavelet transform based image reconstruction scheme is shown in Fig 4.

3. WAVELET RECONSTRUCTION SCHEME

As seen from Fig. 4, the Reconstruction scheme involves three steps:

- 1) N-level wavelet decomposition of input noisy signal.
- 2) Threshold estimation and thresholding of wavelet Coefficients.
- 3) N-level inverse wavelet transform for reconstruction of de noised signal.



Fig. 4: Over All Reconstruction Scheme

Reconstruction using wavelet involves thresholding shown in Fig. 5, in which coefficients having less thresholding value compared to the specific threshold value (λ) are set to zero. This process helps to eliminate noise by keeping original signal characteristics. This is called hard thresholding. Another one i.e soft thresholding set the wavelet coefficients which are above the threshold value as well as simply shrinks or scales other coefficients which are above the threshold value.

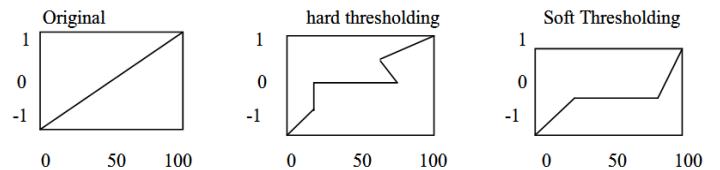


Fig. 5: Soft & Hard Thresholding Methods

There are well known estimation methods available in literature. In this paper, performances of two well known standard threshold estimation methods are investigated for an shepp-logan image and effect of wavelet decomposition level (N) is also computed.

4. SIMULATION RESULTS

A large number of wavelet filters are available, it is difficult to select an optimal mother wavelet for a specific application. The issue is illustrated here with an example that shows the differences between the smoothed images which are obtained by applying to standard phantom three types of mother wavelet filter, Haar, Daub 8, Bio 8 and Daub 15, which have different abilities to filter out high frequency components. The smoothed image are obtained by applying the inverse DWT only to approximation sub band for $j=1,2$ and 3 as shown in Fig 6.

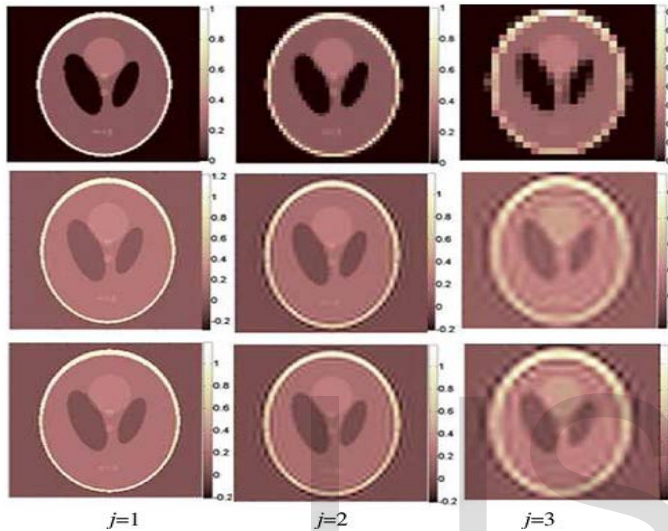


Fig. 6: Wavelet smoothed images for three different mother wavelet filters

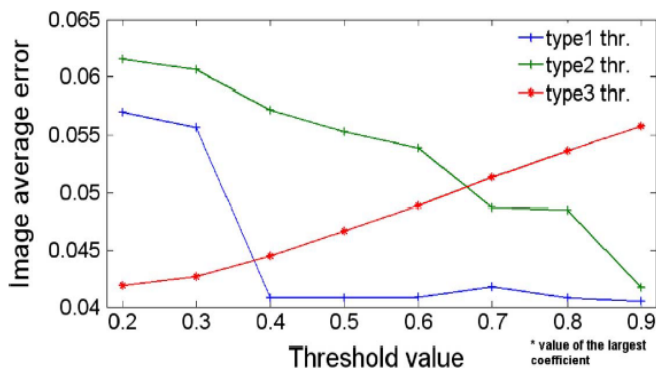


Fig. 7: Effect of different types of thresholding of approximation coefficients on image quality

The image enhancement can be performed simply by selection of the approximation sub band. The coefficients with small magnitude in the approximation sub band mostly correspond to the flat image region while those with higher magnitude effectively represent the image discontinuities. However, those discontinuities are actually represented with a very small number of wavelet coefficients. The types of coefficient selection schemes are investigated namely conventional thresholding, soft thresholding and Hard thresholding as shown in the Fig. 7. To evalu-

ate the effect of thresholding on image quality, we calculated for each type of coefficient selection, the average image error per pixel:

$$average = \frac{\sum_{x,y} |f(x,y) - h(x,y)|}{MN} \quad (3)$$

where, f and h are the reference and reconstructed images respectively with dimensions $M \times N$.

Initially the simulations were run using many wavelet families that were considered to be reasonable stable solution for particular application. They present here with the image reconstruction results with wavelet smoothing using Daub wave filter shown stable solution shown in Fig .8

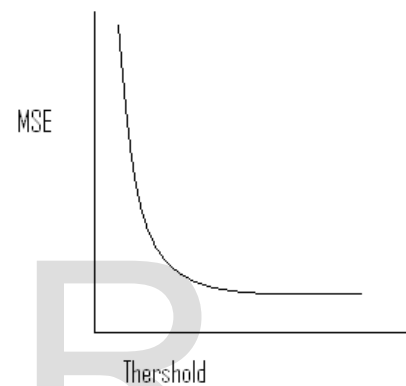


Fig. 8: Image Reconstruction For Daub Filter

5. CONCLUSION

The image reconstruction is one of the important issues to be addressed in developing the tomography. In this paper, the performance of various image reconstruction processes along with different threshold methods is presented. The lower level of decomposition can be preferred. As seen from the result performed by different wavelet filters Daubechies wavelet filter with higher order has better performance than other filters in terms of MSE.

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